

2000 - Higher Paper I

$$1) \sin(p+q) = \sin p \cos q + \cos p \sin q$$

$$OA = \sqrt{6^2 + 8^2} = \sqrt{100} = \underline{\underline{10}}$$

$$OC = \sqrt{12^2 + 5^2} = \sqrt{169} = \underline{\underline{13}}$$

$$\sin p = \frac{8}{10} = \frac{4}{5} \quad \cos p = \frac{6}{10} = \frac{3}{5}$$

$$\sin q = \frac{5}{13} \quad \cos q = \frac{12}{13}$$

$$\therefore \sin(p+q) = \left(\frac{4}{5} \times \frac{12}{13}\right) + \left(\frac{3}{5} \times \frac{5}{13}\right)$$

$$= \frac{48}{65} + \frac{15}{65} = \underline{\underline{\frac{63}{65}}}$$

$$2) a) f'(x) = 3x^2 - 12x + 9 = 0$$

$$(3x-3)(x-3) = 0$$

$$x = \frac{3}{3} = 1 \quad \text{OR} \quad x = 3$$

$$\therefore \underline{\underline{x=1}}$$

$$\text{at } x=1: y = 1^3 - (6 \times 1^2) + 9 \times 1$$

$$= 1 - 6 + 9 = \underline{\underline{4}}$$

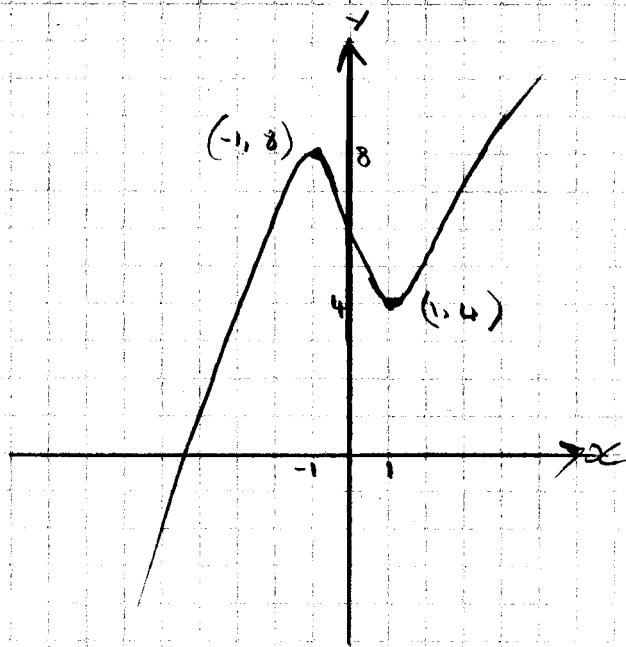
$$\therefore \text{T.P. at } \underline{\underline{(1, 4)}}$$

at T.P

$$\begin{array}{r} 3x^2 - 12x + 9 \\ \underline{-3x^2 + 9x - 9} \\ -3x + 0 \\ \underline{+3x - 9} \\ -9 \end{array}$$

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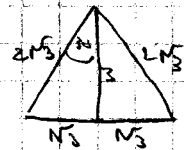
2) b)



c) $4 < R < 8$

3) $M = \tan \theta$

$$M = \frac{2 - -1}{3\sqrt{3} - 0} = \frac{3}{3\sqrt{3}} = \frac{\sqrt{3}}{3}$$



$$\therefore \tan \theta = \frac{\sqrt{3}}{3} \quad \therefore \theta = \tan^{-1} \left(\frac{\sqrt{3}}{3} \right) = \underline{\underline{30^\circ}}$$

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$$4) \quad M = \frac{dT}{dx} \quad M_A = 10x - 15 \\ M_B = 3x^2 - 12$$

$$\text{at } M_A = M_B : \quad 3x^2 - 12 = 10x - 15 \\ 3x^2 - 10x + 3 = 0 \\ (3x - 1)(x - 3) = 0$$

$$\therefore \underline{\underline{x = \frac{1}{3}}} \quad \text{OR} \quad \underline{\underline{x = 3}}$$

at $x = \frac{1}{3}$ the curves are parallel, at $x = 3$ the curves touch and have a common tangent.

$$b) \quad \int_{-1}^3 (x^3 - 12x + 1) - (5x^2 - 15x - 8) dx$$

$$= \int_{-1}^3 x^3 - 5x^2 + 3x + 9 dx$$

$$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} + 9x \right]_{-1}^3$$

$$= \left(\frac{3^4}{4} - \frac{5 \times 3^3}{3} + \frac{3 \times 3^2}{2} + 9 \times 3 \right) - \left(\frac{-1^4}{4} - \frac{5 \times (-1)^3}{3} + \frac{3 \times (-1)^2}{2} + 9 \times (-1) \right)$$

$$= \left(\frac{81}{4} - \frac{135}{3} + \frac{27}{2} + 27 \right) - \left(\frac{1}{4} + \frac{5}{3} + \frac{3}{2} - 9 \right)$$

$$= \left(\frac{243}{12} - \frac{540}{12} + \frac{162}{12} + \frac{324}{12} \right) - \left(\frac{3}{12} + \frac{20}{12} + \frac{18}{12} - \frac{108}{12} \right)$$

$$= \frac{189}{12} + \frac{67}{12} = \frac{256}{12} = \underline{\underline{21\frac{1}{3} \text{ units}}}$$

5) At the limit $L = \frac{b}{1-a}$

$\therefore U_{n+1} = aU_n + 10 \Rightarrow L = \frac{10}{1-a}$

$\therefore U_{n+1} = a^2 U_n + 16 \Rightarrow L = \frac{16}{1-a^2}$

Now $L=L$: $\frac{10}{1-a} = \frac{16}{1-a^2}$

$10(1-a^2) = 16(1-a)$

$10 - 10a^2 = 16 - 16a$

$10a^2 - 16a + 6 = 0 \quad \div 2$

$5a^2 - 8a + 3 = 0$

$(5a-3)(a-1) = 0$

$\therefore a = \frac{3}{5} \text{ or } a = 1$

Now $-1 < a < 1$

$\therefore a = \frac{3}{5}$

$\therefore L = \frac{10}{1-\frac{3}{5}} = \frac{10}{\frac{2}{5}} = \frac{10 \times 5}{2} = \underline{\underline{25}}$

6) For a circle $r > 0$

$\therefore r = \sqrt{g^2 + f^2 - c}$

$2g = 4k$
 $g = 2k$

$2f = -2k \quad c = -k-2$
 $f = -k$

$r = \sqrt{4k^2 + k^2 + k + 2}$
 $= \sqrt{5k^2 + k + 2}$

$\therefore 5k^2 + k + 2 > 0$

\therefore All values of k

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7) $\vec{VK} = \vec{VA} + \vec{AB} + \vec{BK}$

If $\vec{AB} = \begin{pmatrix} 8 \\ 4 \\ 4 \end{pmatrix}$ then $\vec{BK} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$\therefore \vec{VK} = \begin{pmatrix} -7 \\ -13 \\ -11 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}}}$

8) $f(x) = \sin(3x)$ then

$y = \int \sin(3x) = -\frac{1}{3} \cos 3x + C$

at $(\frac{\pi}{9}, 1)$: $1 = -\frac{1}{3} \cos \frac{\pi}{3} + C$

$C = 1 + \frac{1}{3} \cos \frac{\pi}{3}$

$C = 1 + \frac{1}{2} \times \frac{1}{2} = 1 + \frac{1}{4} = \frac{5}{4}$

$\therefore \underline{\underline{y = -\frac{1}{3} \cos 3x + \frac{5}{4}}}$

9) $\log_5 2 + \log_5 50 - \log_5 4$

$= \log_5 100 - \log_5 4$

$= \log_5 25$

$= \underline{\underline{2}}$

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$$\begin{aligned} \cos x - \sin x &= r \cos(x - \alpha) \\ &= r(\cos x \cos \alpha - \sin x \sin \alpha) \end{aligned}$$

$$\cos x - \sin x = r \cos x \cos \alpha + r \sin x \sin \alpha$$

$$r \cos \alpha = 1 \quad r \sin \alpha = -1$$

Squaring and adding: $r^2 \cos^2 \alpha + r^2 \sin^2 \alpha = 1^2 + (-1)^2$

$$r^2(\cos^2 \alpha + \sin^2 \alpha) = 2$$

$$r^2 = 2$$

$$\underline{\underline{r = \sqrt{2}}}$$

NOW: $\frac{r \sin \alpha}{r \cos \alpha} = \frac{-1}{1} = \tan \alpha$

$$\therefore \underline{\underline{\tan \alpha = -1}}$$

Sin is -ve, Cos is +ve $\therefore \alpha$ is in 4th quadrant.

$$\text{R.A.} = \tan^{-1}(1) = \pi/4$$

$$\therefore \alpha = 2\pi - \pi/4 = \underline{\underline{\frac{7\pi}{4}}}$$

$$\therefore \underline{\underline{\cos x - \sin x = \sqrt{2} \cos(x - \frac{7\pi}{4})}}$$

Max at: $\sqrt{2} \cos(x - \frac{7\pi}{4}) = \sqrt{2}$

$$\cos(x - \frac{7\pi}{4}) = 1$$

$$x - \frac{7\pi}{4} = 0$$

$$x = \frac{7\pi}{4}$$

\therefore Maximum value of $\sqrt{2}$ at $x = \frac{7\pi}{4}$

2000 - Higher II

1) at $x=1$: $y = 1 - 3 + 2 = 0$ (1, 0)

$$M = \frac{dy}{dx} = 3x^2 - 6x + 2$$

at $x=1$: $M = 3 - 6 + 2 = \underline{\underline{-1}}$

$$y - b = M(x - a)$$

$$y - 0 = -1(x - 1)$$

$$\underline{\underline{y = 1 - x}}$$

b) $y = y'$

$$x^3 - 3x^2 + 4x = 2x - 4$$

$$x^3 - 3x^2 + 4 = 0$$

2	1	-3	0	4
		2	-2	-4
1	-1	-2	0	

$$(x-2)(x^2 - x - 2) = 0$$

$$(x-2)(x-2)(x+1) = 0$$

$$\therefore \underline{\underline{x = -1}}$$

$$y = 2x - 1 - 4 = \underline{\underline{-6}}$$

$$\therefore \underline{\underline{(-1, -6)}}$$

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2) a) Mid Paret of PQ $(-1, 5)$

$$M_{PQ} = \frac{8}{4} = \underline{\underline{2}}$$

$$\therefore M_{PQ} M_{AB} = -1 \quad M_{AB} = \underline{\underline{-\frac{1}{2}}}$$

$$y - b = M(x - a)$$

$$y - 5 = -\frac{1}{2}(x + 1)$$

$$2y - 10 = -x - 1$$

$$\underline{\underline{2y + x - 9 = 0}}$$

b) $x_c = 1 \quad 2y = 9 - x$
 $2y = 8$
 $\underline{\underline{y = 4}} \quad \therefore C(1, 4)$

$$|CQ| = r = 9 - 4 = \underline{\underline{5}}$$

$$\therefore (x - a)^2 + (y - b)^2 = r^2$$

$$\underline{\underline{(x - 1)^2 + (y - 4)^2 = 25}}$$

c) i) $y = 9$

ii) $2xa + x - 9 = 0$

$$18 + x - 9 = 0$$

$$\underline{\underline{x = -9}}$$

$$\underline{\underline{T(-9, 9)}}$$

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3) a)
$$p(x) = \underline{\underline{3 - \frac{3}{3x}}}$$

b)
$$p(q(x)) = 3 - \frac{3}{\frac{3}{3-x}}$$
$$= 3 - \frac{3(3-x)}{3} = 3 - 3 + x$$
$$= \underline{\underline{x}}$$

4) a) $y = kx(x-4)$

at (2, 4): $4 = 2k(-2)$

$$4 = -4k$$

$$k = -1 \quad \therefore y = -x(x-4)$$

$$y = \underline{\underline{4x - x^2}}$$

b)
$$A = \int_2^{2k} (4x - x^2) dx = \left[2x^2 - \frac{x^3}{3} \right]_2^{2k}$$

$$= \left(2k^2 - \frac{k^3}{3} \right) - \left(8 - \frac{8}{3} \right)$$

$$= \underline{\underline{-\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}}}$$

5) $3 \cos 2x + \cos x = -1$

$$\therefore \cos x = 0.5 \quad \cos x = -0.666$$

$$3(2 \cos^2 x - 1) + \cos x = -1$$

$$x = 60^\circ \quad RA = 48.2$$

$$6 \cos^2 x - 3 + \cos x + 1 = 0$$

$$x = 300^\circ \quad x = 180 - 48.2$$

$$6 \cos^2 x + \cos x - 2 = 0$$

$$= 131.80$$

$$a = 6, b = 1, c = -2 \rightarrow$$

$$x = 180 + 48.2$$

$$= 228.2^\circ$$

$$\therefore \underline{\underline{x = 60^\circ, 131.8^\circ, 228.2^\circ, 300^\circ}}$$

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$$6) A(x) = \frac{3\sqrt{3}}{2} x^2 + 24\sqrt{3} x^{-1}$$

$$\text{Min at } A'(x) = 0 : 3\sqrt{3} x - \frac{24\sqrt{3}}{x^2} = 0 \quad \times x^2$$

$$3\sqrt{3} x^3 - 24\sqrt{3} = 0$$

$$x^3 = \frac{24\sqrt{3}}{3\sqrt{3}}$$

$$x^3 = 8$$

$$\underline{\underline{x = 2}}$$

x	2^-	2	2^+
$A'(x)$	-ve	0	+ve
SHAPE	\	—	/

∴ Min at $x=2$

7) If Perpendicular $a \cdot b = 0$

$$u \cdot v = 2t - 20 + 3t = 0$$

$$5t = 20$$

$$\underline{\underline{t = 4}}$$

$$8) f'(x) = \frac{1}{2}(5x-4)^{-1/2} \times 5$$

$$= \frac{5}{2\sqrt{5x-4}}$$

$$f'(4) = \frac{5}{2 \times \sqrt{16}} = \underline{\underline{\frac{5}{8}}}$$

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9) a) (3, 2, 15)

$$b) \vec{BA} = \begin{pmatrix} 0 \\ 9 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 15 \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \\ -7 \end{pmatrix} \quad \vec{BC} = \begin{pmatrix} 17 \\ 0 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 15 \end{pmatrix} = \begin{pmatrix} 14 \\ -2 \\ -7 \end{pmatrix}$$

$$\cos \theta = \frac{a \cdot b}{|a| |b|}$$

$$|\vec{BA}| = \sqrt{9 + 49 + 49}$$

$$= \sqrt{107}$$

$$|\vec{BC}| = \sqrt{196 + 4 + 49}$$

$$= \sqrt{249}$$

$$\vec{BA} \cdot \vec{BC} = (-3 \times 14) + (7 \times -2) + (-7 \times -7) \\ = \underline{\underline{-7}}$$

$$\therefore \cos \theta = \frac{-7}{\sqrt{107} \times \sqrt{249}} = \underline{\underline{-0.043}}$$

$$\therefore \angle ABC = \cos^{-1}(-0.043) = \underline{\underline{92.5^\circ}}$$

10) $\int (7-3x)^{-2} dx = \frac{1}{-3(-2+1)} (7-3x)^{-2+1} + C$

$$= \frac{1}{3} (7-3x)^{-1} + C$$
$$= \underline{\underline{\frac{1}{3(7-3x)} + C}}$$

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ii) a)
$$P = 0.6Q + 1.8$$

$$M = \frac{1.8}{3} = 0.6$$

b)
$$P = aq^b$$

$$\log_e P = \log_e aq^b$$

$$\log_e P = \log_e a + \log_e q^b$$

$$\log_e P = \log_e a + b \log_e q$$

$$\log_e P = b \log_e q + \log_e a$$

$$P = bQ + \log_e a$$

and $P = 0.6Q + 1.8$

$$\therefore \underline{b = 0.6}$$

$$\log_e a = 1.8$$

$$a = e^{1.8}$$

$$a = \underline{6.05}$$